On Fourier-Galerkin approach for FFT-based homogenization

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In the field of numerical homogenization, the Fourier-based homogenization solvers were introduced by Moulinec and Suquet in their seminal work [1]. Since then, they have established themselves as a compe titive alternative to finite elements in terms of accuracy, efficiency, versatility, and simplicity of implementation. In its basic version, the method works as a fixed-point iterative solution to a periodic Lippman-Schwinger integral equation, whose kernel can be efficiently handled by the Fast Fourier Transform (FFT).

In our recent work [2], we interpreted FFT-based methods in a Galerkin framework that involves the four standard steps, namely (i) introducing a weak form of the governing equations, (ii) projecting the weak form to an approximation space, (iii) applying a numerical quadrature, and (iv) solving the ensuing system of linear equations by a suitable iterative solver.

Specifically, the basic Moulinec-Suquet scheme is obtained when (i) the weak form involves the gradients of the potential, (ii) the approximation space is spanned by trigonometric polynomials, (iii) the trapezoidal rule is employed for numerical integration, and step (iv) involves the Richardson iteration.

The purpose of this talk is twofold: to summarize these developments and to explain how they can be used to develop more efficient FFT-based solvers, considering scalar elliptic problems for simplicity. Specifically, (i) I will focus on a formulation of a new exact integration based discretization technique along with its efficiency to estimate homogenized properties (ii) influence of Krylov subspace methods used to solve non-symmetric rank-deficient linear systems and convergence of an a-posteriori bound on the solution during iterations.

[1] Moulinec, H. and Suquet, P., 1994. A fast numerical method for computing the linear and nonlinear mechanical properties of composites. C. R. Acad. Sci. II, 318(11), pp.1417-1423.

[2] Nachiketa Mishra, Jan Zeman and Jaroslav Vondejc, 2016. A comparative study on low-memory iterative solvers for FFT-based homogenization of periodic media, Journal of Computational Physics , 321, 151-168.