Continuous Computation and Applications

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The classical theory of computation was developed in order to answer Hilbert's Entscheidungsproblem. Most of the pioneering work was done by logicians such as Church, Godel, Kleene, Post and Turing. The model of computation that was developed---the Turing machine (TM)---was not designed for applications to continuous phenomenon associated with some branches of mathematics like analysis and topology. The TM model is inadequate for satisfactorily answering questions such as the computability of the Mandelbrot set or the complexity of Newton’s method for finding roots of polynomials.

Complex dynamics is the study of dynamical systems defined by iteration of analytic functions. Pioneering work in the subject was done in the late 19th century and early 20th century after which the subject remained dormant for decades. The subject has seen a tremendous resurgence in the past 30 years and this is partly due to the availability of high-resolution computer generated pictures of the Mandelbrot set and other fractals. It is somewhat paradoxical that the classical theory of computation cannot answer the question "Is the Mandelbrot set computable?", yet computer generated pictures of the Mandelbrot set have greatly stimulated research in complex dynamics!

In this talk, I shall begin with a historical overview of the subject. I shall then describe one of the possible models of continuous computation called Recursive Analysis. I shall also briefly speak about the Mandelbort set and discuss some of the deep questions in complex dynamics that can be studied using Recursive Analysis.