Convergence of the Reach for a Sequence of Random Manifolds

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Motivated by questions of manifold learning, we study a sequence of random manifolds, generated by embedding a fixed, compact manifold into Euclidean spheres of increasing dimension via a sequence of Gaussian mappings. One of the fundamental smoothness parameters of manifold learning theorems is the reach, or critical radius, of the manifold. Roughly speaking, the reach is a measure of a manifold's departure from convexity, which incorporates both local curvature and global topology. We develop limit theory for the reach of a family of random, Gaussian-embedded, manifolds, establishing both almost sure convergence for the global reach, and a fluctuation theory for both it and its local version. The global reach converges to a constant well known both in the reproducing kernel Hilbert space theory of Gaussian processes, as well as in their extremal theory. Joint work with Robert J. Adler, Jonathan E. Taylor, and Shmuel Weinberger.