

The Scaling Method of Pinchuk

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The Riemann mapping theorem asserts that any proper simply connected domain in the complex plane is biholomorphic to the unit disk. This theorem fails spectacularly in higher dimensions. Nevertheless, there are several results that can be thought of as generalizations of the Riemann mapping theorem. One such result is the celebrated result of Wong, which was later generalized by Rosay. A special case of Wong's result states that any smoothly bounded convex domain in \mathbb{C}^n which has a non-compact automorphism group is biholomorphic to the unit ball.

Wong's original proof, as well as the proof of Rosay's refinement, are quite involved. However, Pinchuk has given a simple proof of the Wong-Rosay theorem using a method he discovered now known as the scaling method. The scaling method is very powerful and has been extensively used in the literature on holomorphic mappings of several complex variables.

I will try to illustrate the basic ideas behind the scaling method by giving a brief sketch of the proof of the Wong-Rosay theorem in the complex plane. Only a knowledge of complex analysis in one variable will be needed to understand the talk.