Formal Language Theory

Problem Sheet 1

1. Find Regular Grammars for the following languages on \( \{a, b\} \)
   (a) \( L = \{ w : n_a(w) \text{ and } n_b(w) \text{ are both even} \} \).
   (b) \( L = \{ w : (n_a(w) - n_b(w)) \mod 3 = 1 \} \).
   (c) \( L = \{ w : (n_a(w) - n_b(w)) \mod 3 \neq 0 \} \).

2. Find a regular grammar that generates the set of all Pascal real numbers.

3. Find the minimal DFA for the following languages
   (a) \( L = \{ a^n b^m : n \geq 2, m \geq 1 \} \).
   (b) \( L = \{ a^n b^m : n \geq 0 \} \cup \{ b^n a : n \geq 1 \} \).
   (c) \( L = \{ a^n : n \geq 0, n \neq 3 \} \).

4. Find regular expression for the set \( \{ a^n b^m : (n + m) \text{ is even} \} \).

5. Give a regular expression for the following languages:
   (a) \( L = \{ a^n b^m : n \geq 4, m \leq 3 \} \).
   (b) \( L = \{ a^n b^m : n \geq 1, m \geq 1, nm \geq 3 \} \).
   (c) \( L = \{ ab^n w : n \geq 3, w \in \{a, b\}^+ \} \).
   (d) \( L = \{ w \in \{0, 1\}^* : w \text{ has exactly one pair of consecutive zeros} \} \).
   (e) \( L = \{ w \in \{0, 1\}^+ : w \text{ ends with 01} \} \).
   (f) \( L = \{ w \in \{0, 1\}^+ : |w|_0 \text{ is even} \} \).

6. Prove the following:
   (a) \( (r_1^*)^* \equiv r_1^* \).
   (b) \( r_1^* (r_1 + r_2)^* \equiv (r_1 + r_2)^* \).
   (c) \( (r_1 + r_2)^* \equiv (r_1^* r_2^*)^* \).
   for all regular expression \( r_1 \) and \( r_2 \). Here \( \equiv \) stands for equivalence in the sense of the language generated.

7. Find an NFA that accepts the language \( L(aa^*(a + b)) \).

8. Find DFA that accepts the following languages:
   (a) \( L(aa^* + aba^*b^*) \).
   (b) \( L(ab(a + ab)^* + (a + aa)) \).
   (c) \( L((aab)^* + (aaa^* + b)^*) \).
   (d) \( L(((aa^*)^*b)^*) \).

9. Construct a DFA that accepts the language generated by the grammar

\[
S \rightarrow abA \\
A \rightarrow baB \\
B \rightarrow aA/bb
\]
10. Construct right and left linear grammars for the following language:

\[ L = \{a^n b^m : n \geq 2, m \geq 3\} \]

11. Construct a right linear grammar for the following language:

\[ L((aab^*ab)^*) \]