NEW SPECTRAL THEOREM FOR VECTOR SPACES OVER FINITE FIELDS $\mathbb{Z}_p$

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The study of vector spaces over fields of characteristic zero is very old. Several classical results have been established in this direction. One of the major results is the spectral theorem in case of inner product spaces. But the study of vector spaces over finite fields $\mathbb{Z}_p$, $p$ a prime is in fact missing or very meager in mathematical literature. This paper aims to study only vector spaces over $\mathbb{Z}_p$ where $\mathbb{Z}_p$ is the prime field of characteristic $p$. Many interesting results about these vector spaces are obtained in this paper. We call these vector spaces as F-vector spaces. The two main contributions are

1. Special Theorem for F-vector spaces.
2. Conditions for the existence of n-linearly independent vectors where n is the dimension of the F-vector space over $\mathbb{Z}_p$.

To have spectral theorem we need the concept of standard inner product. In our F-vector spaces we can have $(a/a) = 0$ without $a$ being equal to zero. Hence we define F-inner product on these F-vector spaces. Using these F-inner product we are able to prove the spectral theorem for the F-vector
spaces. The other major interesting result is that we are able to prove: if \( V \) is a \( \mathbb{F} \)-vector space of dimension \( n \) and \( T \) a linear operator on \( V \) resulting in a \( n \times n \) diagonal matrix, then this \( T \) has \( n \) distinct eigen vectors only when at least 2 of the diagonal entries are distinct. Several other interesting and important properties about these \( \mathbb{F} \)-vector spaces are also obtained in this paper.