A NEW TYPE OF CO-SUBRING RELATION IN RINGS

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In this paper we define a new notion called cosubring relation in rings, which are analogous to coset relation in groups. The reason for defining cosubrings is that we can say in groups whether any two elements are related or not via a proper subgroup, but in case of rings we are not able to achieve this, if both the operation in a ring are to be given equal importance.

These cosubring has distinct properties which are not true in case of cosets in groups. Cosubrings do not partition the ring $R$ and there does not exist a one to one correspondence between two cosubrings of a ring $R$. This distinct behaviour of cosubrings has
interested us to study these structures and obtain some interesting results in this direction.

We relate two elements of a ring, using these cosubring structure, via a proper subring, which is not an ideal. With this cosubring relation we prove that if $R$ is a ring with unity, a pair of elements $x, y \in R$ has cosubring relation, then either the subring $S \subset R \setminus \{x, y\}$ has a unit element or otherwise $x$ or $y$ is a zero divisor. Thus this cosubring relation gives a condition on the ring to have either a unit element or have zero divisors. So in case of the group ring $K \mathbb{G}$ where $G$ is a torsion free non-abelian group and $K$ any field of characteristic zero, the study of the cosubring relation leads to an equivalent formulation of the six decade old conjecture in group rings viz. : Does the group ring $K \mathbb{G}$ have non trivial zero divisor or equivalently does there exist a proper subring of the group ring $K \mathbb{G}$ for which a pair of elements is cosubring related?
Finally this cosubring relation can be made into an equivalence relation under certain specified conditions.