MODULARITY AND QUASI-MODULARITY IN LOOPS

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In this paper we study conditions on the loops to have subloops which forms a modular or quasi modular lattices. In fact we prove that all subloops of the loop \( \{\mathbb{Z}_n(m), \cdot\} = \{e, 1, 2, 3, \ldots, n / m < n, n > 3, n - \text{prime}, (m - 1, n) = 1 \text{ and } (m,n) = 1 \} \) with the operation \( \cdot' \) \( e \cdot' r = r \cdot' e = r, r \cdot' s = ms - (m - 1) r, (\text{mod } n) \) for all \( r, s \in \mathbb{Z}_n(m), r \neq s, r \neq e, \text{ and } s \neq e \) and \( r \cdot' r = e \) form a modular lattice of the special form \( \mathbb{M}_n \). None of these subloops form a distributive lattice. Further none of the subloops are normal subloops except in the case when \( \mathbb{Z}_n(m) \) is a commutative loop. This is a marked difference between the groups and loops for in case of groups only normal subgroups form a modular lattice but in case of this class of loops though none of the subloop is normal, yet the collection of subloops form a modular lattice. A study of quasi-modularity is also dealt in the case of these loops.