GROUPOID RINGS

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In this paper, we introduce the notion of groupoid rings, which are analogous to semigroup rings or group rings. In this paper instead of using a semigroup or a group, we use groupoids where groupoids are nothing but non-associative semigroups; $G_m = \{g_0, g_1, \ldots, g_m\}, 0 \leq m \leq \alpha$ with a binary operation ‘$\circ$’ defined by $g \circ g = g_{(t+sg) \mod m}; t$ and $s$ are integers chosen such that $(t, s) = 1$ and $1 < t < m$ and $1 < s < m$. Clearly $(G_m, \circ)$ is a groupoid. Further the operation in general is non-associative. We in this paper, choose rings over which groupoid rings are defined, as $\mathbb{Z}$ or $\mathbb{Q}$ or $\mathbb{R}$ only. Thus in this paper by $\mathbb{Z}G_m$, or $\mathbb{Q}G_m$ or $\mathbb{R}G_m$, we mean the groupoid rings of the groupoid $G_m$ over $\mathbb{Z}$ or $\mathbb{Q}$ or $\mathbb{R}$ respectively. We see $G_m \subseteq \mathbb{Z}G_m$, $G_m \subseteq \mathbb{Q}G_m$ and $G_m \subseteq \mathbb{R}G_m$ as $1 \in (\mathbb{Z}, \mathbb{Q} \text{ and } \mathbb{R})$. But in general $\mathbb{Q} \subseteq \mathbb{Q}G_m$, this can happen only when $1 \in G_m$. This is marked difference between groupoid rings and group rings. Further groupoid rings are non-associative. Thus for every $m, 1 < m < \alpha$, we get a class of non-associative rings $\mathbb{Z}G_m$, $\mathbb{Q}G_m$ and $\mathbb{R}G_m$. This method provides us a means to get non-abstract non-associative rings. Many other interesting properties about these structures are studied.