PROPERTIES OF GROUP SEMIRINGS WHEN SEMIRINGS ARE FINITE DISTRIBUTIVE LATTICES

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In this paper, we introduce and study groups over finite semirings. These finite semirings $S$ are taken only as distributive lattices. Throughout this paper $G$ will denote a group under multiplication and $SG$ is the group semiring of the group $G$ over the semiring $S$. Clearly $G \subseteq SG$ and $S \subseteq SG$ as both $G$ and $S$ contain the identity 1. We obtain several interesting results about these structures. Clearly the group semiring $SG$ is not a distributive lattice. We prove the following results.

1. If $G$ is a finite group, then the group semiring $SG$ has non-trivial zero-divisors and non-trivial idempotents only which $S$ is not a chain lattice.

2. If $S$ is a chain lattice, the group semiring $SG$ has only non-trivial idempotents and no zero-divisors, which is a marked difference between group rings and group semirings.

If $G$ is a Torsion-free abelian group and $S$ is a finite chain lattice, then $SG$ is a semifield. These group semirings contain elements of
the form \( x \cdot y = g \) where \( g \) is the group element and \( x \) and \( y \) are elements from \( SG \setminus G \). Here we obtain condition on \( S \) and \( G \) so that we are guaranteed of such types of elements. For example the group semiring \( SG \) has units, then at least \( S \) must have a sublattice whose homomorphic image is isomorphic to a Boolean Algebra of order 4 and \( G \) must have at least elements of order 2.