SYMME TRIC GROUP SEMIRINGS

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In this paper, we introduce the notion of group semi-rings, which are analogous structure to group rings. Here we study only symmetric group $S_n$ of degree $n$ over a commutative semi-ring with unity. Throughout this paper, $KS_n$ denotes the symmetric group semi-ring of the symmetric group $S_n$ over the semi-ring $K$. We prove when $K$ is the semi-ring of positive integers with zero $Z^+ \cup \{0\}$, or the set of positive rational with zero $Q^+ \cup \{0\}$ or the set of positive real numbers with zero $R^+ \cup \{0\}$ and $S_n$, the symmetric group semi-ring has no zero-divisors leading to the non-existence of non-trivial idempotents and non-trivial nilpotents. This result is a complete deviation from the theory of group rings for the case of the symmetric group rings; the group ring contains non-trivial zero-divisors, non-trivial idempotents and non-trivial nilpotents. Further, we are able to obtain many non-commutative division semi-rings when we use symmetric groups over $Z^+ \cup \{0\}$ or $Q^+ \cup \{0\}$ or $R^+ \cup \{0\}$. Now, when we replace the above said semi-rings by a finite semi-ring viz. a finite distributive lattice $L$, we are able to show that symmetric group semi-ring $LS_n$ contains zero-divisors, idempotents and nilpotents. We are able to prove when symmetric group semi-rings give either non-commutative semi-rings or division semi-rings depending on whether we take a distributive lattice or $Z^+ \cup \{0\}$ or $Q^+ \cup \{0\}$ or $R^+ \cup \{0\}$.