CODED SEMIVECTOR SPACES

W.B.Vasanthan Kandasamy and V.Vaithiyanathan

Code loops were first discovered by R. A. Parker and are well known in literature as Parker loops (named after the discoverer). A coded semivector space (or CVS) is defined as a 4-tuple \((C, \sigma, \chi, \alpha)\) where \(C\) is a finite dimension vector space over \(F_p\) and \(\sigma: C \rightarrow Z\) (\(Z\) the group of order \(p\)). \(\chi: C \times C \rightarrow Z\) and \(\alpha: C \times C \times C \rightarrow Z\) satisfying

\[
\sigma(c^n) = \sigma(c)^n
\]

\[
\sigma(cd) = \begin{cases} 
\sigma(c)\sigma(d)\chi(c, d), & p = 2 \\
\sigma(c)\sigma(d) & \text{for } p > 2
\end{cases}
\]

\[
\begin{align*}
\chi(c, c) &= 1 \\
\chi(c, d) &= \chi(d, c)^{-1} \\
\chi(c^n, d) &= \chi(d, c)^n \\
\chi(cd, e) &= \chi(c, e)\chi(d, e)\alpha(c, d, e)^3 \\
\alpha(c, d, d) &= \alpha(d, c, d) = \alpha(d, d, c) = 1 \\
\alpha(c, d, e) &= \alpha(d, c, e)^{-1} = \alpha(d, e, c) \\
\alpha(c^n, d, e) &= \alpha(c, d, e)^n \\
\alpha(cd, e, f) &= \alpha(c, e, f)\alpha(d, e, f)
\end{align*}
\]

for all \(c, d, e, f \in C\) for all \(n \in Z\).

All Rights Reserved. This work is Copyright © W.B.Vasanthan Kandasamy and V.Vaithiyanathan, 2003. Mathematicians can use the above material for research purposes, but the work of the author(s) must be acknowledged. Violators of copyright, and those indulging in plagiarism and intellectual theft are liable for strict prosecution.

e-mail: vasantha@iitm.ac.in
web: http://mat.iitm.ac.in/~wbv
A loop $L$ is a coded extension of $C$ if $L$ satisfies the conditions $L$ has a central subgroup $Z$ of order $p$ such that $L/Z \cong C$. Let $\gamma, \delta, \eta \in L$ denote the pre images of $c, d, e \in C$ respectively. Then

$$\gamma^p = \sigma(c)$$
$$[\gamma, \delta] = \chi(c, d)$$
$$[\gamma, \delta, \eta] = \alpha(c, d, e)$$

where the values of $\sigma$, $\chi$ and $\alpha$ are taken to be in the central subgroup $Z$. We define analogously the coded semivector space by chiefly replacing $C$ by a finite dimensional semivector space and show the coded extension in general is a groupoid.